Docket No. 1999-0045 (2455-4601)

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

UTILITY APPLICATION AND FEE TRANSMITTAL (1.53(b))

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BOX PATENT APPLICATION
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Sir:

Transmitted herewith for filing is the patent application of

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For: METHOD FOR ESTIMATING TIME AND FREQUENCY OFFSET IN AN OFDM SYSTEM

Enclosed Are:

20 page(s) of specification
1 page(s) of Abstract
11 page(s) of claims
11 sheets of ☒ Formal ☐ Informal drawings

_____ page(s) of Declaration and Power of Attorney

☐ Unsigned
☐ Newly Executed
☐ Copy from prior application
☐ Deletion of inventors including Signed Statement under 37 C.F.R. §1.63(d)(2)

☐ **Incorporation by Reference**:

☐ The entire disclosure of the prior application, from which a copy of the combined Declaration and Power of Attorney is supplied herein, is considered as being part of the disclosure of the accompanying application and is incorporated herein by reference.

☐ Microfiche Computer Program (Appendix)

- ☐ page(s) of Sequence Listing
- ☐ computer readable disk containing Sequence Listing
- ☐ Statement under 37 C.F.R. §1.821(f) that computer and paper copies of the Sequence Listing are the same

☐ Assignment Papers (assignment cover sheet and assignment documents)

- ☐ A check in the amount of $40.00 for recording the Assignment
- ☐ Charge the Assignment Recordation Fee to Deposit Account No. 13-4503, Order No.

- ☐ Assignment Papers filed in the parent application Serial No.

☐ Certification of chain of title pursuant to 37 C.F.R. §3.73(b)

☐ Priority is claimed under 35 U.S.C. §119 for:
Application No(s). _____ , filed _____, in _____ (country).

- ☐ Certified Copy of Priority Document(s) [ ]
  - ☐ filed herewith
  - ☐ filed in application Serial No. _____, filed _____.

- ☐ English translation document(s) [ ]
  - ☐ filed herewith
  - ☐ filed in application Serial No. _____, filed _____.

☐ Priority is claimed under 35 U.S.C. §119(e) for:
Provisional Application No. _____, filed _____.

☐ Priority is claimed under 35 U.S.C. §120 for:
Application No(s). _____, filed _____, in _____.

☐ Information Disclosure Statement

- ☐ Copy of [ ] cited references
- ☐ PTO Form-1449
- ☐ References cited in parent application Serial No. _____, filed _____.

☐ Preliminary Amendment

☒ Return receipt postcard (MPEP 503)

☐ This is a ☐ continuation ☐ divisional ☐ continuation-in-part of prior application serial no. __, filed _____.

- ☐ Cancel in this application original claims _____ of the parent application before calculating the filing fee. (At least one original independent claim must be retained for filing purposes.)
- ☐ A Preliminary Amendment is enclosed. (Claims added by this Amendment have been properly numbered consecutively beginning with the number following the highest numbered original claim in the prior application.

☐ The status of the parent application is as follows:

☐ A Petition for Extension of Time and a Fee therefor has been or is being filed in the parent application to extend the term for action in the parent application until _____.

☐ A copy of the Petition for Extension of Time in the co-pending parent application is attached.

☐ No Petition for Extension of Time and Fee therefor are necessary in the co-pending parent application.

☐ Please abandon the parent application at a time while the parent application is pending or at a time when the petition for extension of time in that application is granted and while this application is pending has been granted a filing date, so as to make this application co-pending.

☐ Transfer the drawing(s) from the parent application to this application

☐ Amend the specification by inserting before the first line the sentence: This is a continuation of co-pending application Serial No. _____, filed _____.

I. CALCULATION OF APPLICATION FEE				
	Number Filed	Number Extra	Rate	Basic Fee $760.00/380.00
Total Claims	82- 20 =	62x	$18.00/$9.00	$ 1,116.00
Independent Claims	4- 3 =	1x	$78.00/$34.00	$ 78.00
☐ Multiple Dependent Claims		If marked, add fee of $260.00 ($130.00)		$ 0.00
			TOTAL:	$ 1,954.00

☐ A statement claiming small entity status is attached or has been filed in the above-identified parent application and its benefit under 37 C.F.R. §1.28(a) is hereby claimed. Reduced fees under 37 C.F.R. §1.9 (f) paid herewith $_____.

☐ A check in the amount of $ _____ in payment of the application filing fees is attached.

☒ Charge fee to Deposit Account No. 13-4503 Order No. 2455-4601. A DUPLICATE COPY OF THIS SHEET IS ATTACHED.

☒ The Assistant Commissioner is hereby authorized to charge any additional fees which may be required for filing this application pursuant to 37 CFR §1.16, including all extension of time fees pursuant to 37 C.F.R. § 1.17 for maintaining copendency with the parent application, or credit any overpayment to Deposit Account No. 13-4503 Order No. 2455-4601. A DUPLICATE COPY OF THIS SHEET IS ATTACHED.

Respectfully submitted,
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TITLE OF THE INVENTION

METHOD FOR ESTIMATING TIME AND FREQUENCY OFFSET IN AN OFDM SYSTEM

This patent application is related to U.S. Serial No. 09/128,738 filed August 5, 1998 by Alamouti, Stolarz, & Becker entitled "Vertical Antenna Adaptive Array", and U.S. Serial No. 08/796,584 by Alamouti et al., entitled "Method for Frequency Division Duplex Communications," assigned to AT&T Wireless Services and incorporated herein by reference.

Field of the Invention

The invention relates to a method to synchronize a multicarrier transmission system.

Background of the Invention

Synchronization techniques for OFDM have been extensively studied: obtaining good performance under a variety of channel conditions with minimal signal processing is challenging. A good OFDM synchronization technique will be applicable to more than the wireless high speed data communications system currently being studied - OFDM is being used or being considered in a variety of LEC networks in the form of ADSL, in Digital Audio Broadcast systems, in cable modems, and in digital television systems. OFDM is a special case of multicarrier transmission systems. The techniques described herein are generally applicable to other forms of multicarrier systems, e.g., discrete multitone (DMT) systems.

SUMMARY OF THE INVENTION

The invention uses inherent characteristics of the frequency domain representation of the data symbols. By computing a differential-in-frequency function across a large number of OFDM tones, robust estimates of time and frequency offset can be easily obtained. The technique also allows the system designer to directly trade performance in the presence of channel impairments against signal processing complexity. Analysis and simulation have shown good performance in the presence of noise and channel delay dispersion, impairments that are the harshest in a wireless environment.

Prior techniques for OFDM synchronization have focussed on the time domain representation of the signal. Those that have recognized the translation of time and frequency offset to the frequency domain have apparently not considered the systematic modification of the signal by the offsets.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 is a block diagram of OFDM transmitter.

FIG. 2 is an OFDM block structure.

FIG. 3 is a block diagram of OFDM receiver.

FIG. 4 depicts a plot of phase vs. tone number, no timing offset.

FIG. 5 depicts a plot of phase vs. tone number, 1 sample timing offset.

FIG. 6 shows differential in frequency constellation with 10 sample timing offset.

FIG. 7 shows the 4^{th} power of differential in frequency constellation.

FIG. 8 shows the phase vs. tone number with $1/10^{th}$ tone frequency offset.

FIG. 9 shows the timing offset constellation with 6 dB SNR and 10 sample timing offset.

FIG. 10 shows differential in frequency constellation with ½ tone frequency offset and 22 samples timing offset.

FIG. 11 shows the effect of noise on individual tone phase.

DISCUSSION OF THE PREFERRED EMBODIMENT

I. INTRODUCTION

The problem of joint timing and frequency offset estimation is critical for the proper operation of a digital transmission system. It is also a difficult problem when it comes to system implementation. The problem is compounded by the fact that timing offset may often create similar looking signal impairments to frequency offset. A number of prior studies have

attempted to find timing and frequency offset estimation techniques that are robust in the face of a wide variety of impairments present on a wireless channel.

We focus on the problem of joint timing and frequency offset estimation for an OFDM system. We first present a brief overview of an OFDM system. We review prior work in the area and describe our system's requirements as an specific example of OFDM system design. We conclude by showing the performance of the method in the presence of a variety of channel impairments and describing practical limitations of the technique.

II. OFDM OVERVIEW

Delay spread, that is the time dispersion of a brief transmitted pulse, is a major impediment to high speed data transmission in the outdoors, high mobility wireless environment. Similarly, signal reflections in the landline telephone copper cable plant or in a TV cable systems coaxial cable plant create similar dispersion and make high speed communications over the local access network difficult. In a wireless system, distant signal reflectors can create several microseconds of delay spread. The high mobility (i.e., high vehicle speeds) environment will create a rapidly changing channel. Adaptive channel equalization techniques have been used to combat some of this effect, but are limited in their ability to deal with large amounts of rapidly changing delay spread, especially when the system is transmitting at the high symbol rates needed to attain high throughput. Conversely, using low symbol rates to mitigate channel dispersion will require dense constellations in single carrier systems. Dense signal constellations will be subject to degradation due to noise, interference, and fading.

In an OFDM system, a single higher speed bit stream is separated into a multiplicity of slower speed bit streams, each of the slower bit streams used to modulate one of a set of carriers. The carriers are chosen within a single bandwidth and, for an OFDM system, with their inter-

tone spacing chosen to insure orthogonality of the resulting combined waveforms. Generic multicarrier systems are not bound by the strict othogonality constraints but, for ease of implementation will often use very similar techniques to the OFDM system described herein. By designing the transmission in this way, the benefit of low symbol rates can be attained without the penalty of dense constellations – each individual carrier has a sparse constellation, often QPSK. The total capacity of the OFDM system is determined by the union of a large number of these individually sparse constellations. Typically, the OFDM transmitter would be implemented by generating a series of complex numbers, representing the phase of the individual tones, and using the Inverse Discrete Fourier Transform (IDFT) to convert the series of tones into a time domain waveform. For practical reasons, in order to make the analog reconstruction and antialiasing filters realizable, the size of the IDFT (or more often the Inverse Fast Fourier Transform (IFFT)) is usually larger than the number of tones. In addition, extra samples are prepended and appended to the transformed waveform. These "cyclic prefix" and "cyclic suffix" samples make the transmitted signal more robust against time dispersion and timing offset – as long as the received signal, plus its various delayed copies, are sampled within the cyclic extension, the constant amplitude of the received spectrum will be preserved. At the transmitter, the summation of a large number of sinusoids can generate a signal with a large peak-to-average waveform. A peak control function is used to minimize this effect, relaxing the requirements on RF amplifier stages. FIG. 1 shows a simplified block diagram of an OFDM transmitter.

Modulated tones occupy only the first group of input tones to the IFFT (about one third of the total number of tones for the OFDM system here). The rest of the IFFT inputs are set to zero, insuring that the output waveform has no signal energy at higher frequencies and thus oversampling the resulting waveform. As stated above, this simplifies the requirement for

antialiasing filtering of the final waveform. Besides this oversampling, other redundant information is added to the OFDM waveform – the IFFT waveform samples are cyclically extended both before and after the desired set of samples. For an N point IFFT, the cyclically extended suffix samples are simply copied from the beginning of the waveform as (In the equations here, upper case letters indicate signal representations in the frequency domain. Lower case letters indicate the time domain):

$$x_{i+N} = x_i$$

The cyclic prefix is computed similarly, copying the samples at the end of the waveform to the front of the extended waveform.

Two other segments that are added to the IFFT waveform are the windowed section and the guard interval. The guard interval is a period during which the transmitted samples are all zero. At the expense of a slightly reduced transmission efficiency, the presence of this guard interval helps to insure that the samples received during a given OFDM block are not contaminated with delayed samples from a previous block. Since the OFDM system described here will be used in packet mode, it is expected that the transmitter will be turned on and off during each OFDM burst. To prevent the radiated signal from "splattering" outside of its assigned channel, "windowing" of the OFDM samples is used. By shaping the transmitter ramp up and ramp down with a raised cosine pulse shape, the system bandwidth is confined to little more than the bandwidth occupied by the set of OFDM tones alone. Windowing also reduces the system's sensitivity to frequency offset and doppler by reducing the amplitude of the interference contribution from adjacent tones. FIG. 2 OFDM block structure illustrates these items and provides the particular parameters used for the system described here.

The OFDM receiver structure mirrors the operation of the OFDM transmitter. While the technique described here is applicable to a multicast network, that is, a network with one transmitter and several independent receivers, to simplify the description a simple one transmitter, one receiver network is presented. It can readily be seen that each receiver in a multicast network could individually perform the operations described on a common transmitted signal. In the receiver, extra samples are stripped from the received waveform and the resulting waveform is transformed into the frequency domain. Here, assuming that there was no frequency or timing offset in the time domain waveform, a series of tones are processed, with each conveying one of the lower speed bit streams. If QPSK were used at the OFDM transmitter to modulate each tone, then each OFDM tone burst would convey 2 bits of information per tone in the phase of that tone. A simplified OFDM receiver is shown in FIG. 3.

While the relatively long symbol period of the OFDM waveform makes the system resistant to delay spread, incorrect timing phase or frequency offset in the received signal will quickly degrade system performance. Most of the extra samples in the time domain waveform insure that the amplitude of the received OFDM spectrum remains constant in the presence of delay spread and timing offset, due to the cyclic shifting property of the Fourier Transform. However, information is conveyed in the *phase* of the received tones, so the damaging effects of timing offset must be corrected.

Frequency offset has a different effect on the received waveform. While the OFDM transmitter generated orthogonal tones by creating a specific linear harmonic relationship between all the tones, frequency offset shifts each tone by an equal amount, creating an affine relationship between tone frequencies, thus destroying the linear harmonic structure and creating inter-tone interference. Like inter-symbol interference in a single carrier system, frequency

offset allows energy from adjacent tones to "bleed" into a desired tone, reducing system performance.

III. SYSTEM PARAMETERS

As an example of a practical system, the OFDM system here is intended to provide peak end user data rates of up to 384 kb/s. As shown in FIG. 2, this system transmits OFDM bursts with a duration of 288.462 μsec. The 512 FFT samples represent the transform of 189 discrete complex tones. These 189 tones are spaced every 4.232 kHz (skipping the three tones nearest the center frequency), so the total bandwidth is 812.5 kHz. With QPSK modulation on each complex tone, this means that the raw channel rate is 1.3104 Mb/s. With a rate ½ Reed-Solomon code plus framing and control overhead, the peak end user capacity of 384 kb/s is easily attained

IV. PRIOR WORK

Schmidtl's and Cox's papers in the IEEE Transactions on Communications and in the Proceedings of the International Communications Conference define techniques for estimating time and frequency offset based on signal processing in the time domain, using 'pilot' signals. While the pilot signals are useful for establishing a reference for calculating synchronization parameters, they require that transmit energy be expended and signal bandwidth be consumed that could have been used to transmit end user information.

Moose's technique similarly requires repetition of an OFDM block, essentially using one of the repetitions as a 'pilot' signal. Frequency offset is computed by comparing samples of the first block to the second in the time domain. This technique is only used for frequency offset estimation. It does not address the need for time synchronization.

The work of Pollet, et. al., describes the degradation in system performance when adequate timing synchronization is not used.

Finally, van der Beek, et. al., describe a timing synchronization technique that relies on inherent redundancy in the OFDM time domain waveform. Because the cyclic extension of the FFT samples are simply ordered copies of other samples in the waveform, this technique relies on computing the time correlation between the repeated samples to estimate the timing offset. [J.-J. van de Beek, M. Sandell, M. Isaksson, and P. Borjesson, "Low-complex frame synchronization in OFDM systems," in Proc. ICUPC, Nov. 1995, pp. 982-986.] There are relatively few cyclically extended samples in the OFDM block, so the robustness to noise may be somewhat limited, processing a single OFDM block.

None of the previous techniques process the signals in the frequency domain as the current scheme does, and none recognize the inherent structure of the frequency domain signals that this scheme relies on. Except for van der Beek's scheme, all require the use of redundant pilot signals.

V. APPROACH

Frequency offset causes each tone in the OFDM cluster to be shifted in frequency by the same amount. The complex OFDM time domain waveform appears to be rotated on a time-sample by time-sample basis by a continuously rotating phasor. While this effect is instructive to understand how to *undo* the effect of frequency offset, there is nothing immediately obvious about the time domain waveform that suggests how the frequency offset can be estimated. Timing offset similarly shifts all samples by a fixed time interval which, again by itself, gives little information that allows estimation of the amount of offset when observing the time domain waveform. The current method performs estimation of the time and frequency error in the

frequency domain, unlike previous approaches which did their estimation in the time domain. In addition to realizing the other advantages described herein, a frequency domain approach to synchronization simplifies the architecture of the receiver.

TIMING OFFSET CORRECTION

First, consider the effect of timing offset on the OFDM tones. FIG. 4 depicts a plot of the phase versus frequency (here, frequency corresponds to FFT output bin number) for a randomly generated OFDM tone cluster with no timing offset. It can be seen that each tone takes on one of a discrete set of phases (one of four, in this case, since the tones are QPSK modulated). Each tone is independently modulated, so the transitions between the phase of $tone_i$ and $tone_{i+1}$ are random multiples of $\pi/2$. For this, and the subsequent phase plots, the FFT size is 512 and there are 189 active tones.

In contrast, Figure 5 depicts a plot of phase versus frequency where the OFDM signal has been delayed by one time sample (462 μsec in the example system). It can be seen that the tone modulation is still present, however, each tone has a phase that is slightly offset from the previous one. As shown in the section of this memo that presents the analysis of the effects of timing offset, this is exactly what is expected.

To understand how timing offset has changed the received signal, first consider an OFDM waveform x_n, generated from a series of OFDM tones, X_m:

$$x_n = \frac{1}{\sqrt{N}} \cdot \sum_{m=0}^{N-1} X_m \cdot e^{j \cdot \frac{2 \cdot \pi \cdot n \cdot m}{N}}$$

At the OFDM receiver, the waveform is assumed to have a timing offset, Δt. By the shifting property of the Fourier Transform, a time delayed signal in the time domain

$$r(t) = x(t - \Delta t)$$

Has as its transform:

$$R(f) = X(f) \cdot e^{-j \cdot 2 \cdot \pi \cdot f \cdot \Delta t}$$

So, as demonstrated in FIG. 5, it can be seen that the time delay has caused a phase rotation in the frequency domain that increases linearly with frequency. Consider how the Discrete Fourier Transform of the sampled receive signal is affected by timing offset:

$$R_n = X_n \cdot e^{j \cdot \frac{2 \pi \cdot n \cdot \Delta t}{N}}$$

For any adjacent pair of tones, R_i and R_{i+1},

$$\Delta t = \frac{N}{2 \cdot \pi} \cdot (\arg(R_i) - \arg(R_{i+1}))$$

This suggests an approach that could be used to estimate the amount of timing offset: measure the differential phase from one tone to the next and adjust the sampling point to compensate. It is instructive to note that as shown in FIG. 5 and in the equation above, timing offset has created the same phase difference between every pair of tones: R_i and R_{i+1}. This means that, while noise and other impairments may perturb the individual tone phases, there is a systematic change in phase from tone to tone due to timing offset. By estimating timing offset across many or all the adjacent tone pairs in a burst, it is feasible to accurately estimate the timing offset in the presence of a collection of other impairments: frequency offset, noise, fading, delay spread/frequency selective fading, etc. Particular performance results are presented in the next section. Of course, the same inherent characteristics that cause each adjacent pair of tones to exhibit the identical phase difference also cause each tone pair separated by N other tones to exhibit a phase difference that is different than that between adjacent tones, but also identical to all other N tone separated tone pairs. Further, if it is necessary for the receiver to avoid using certain tones for phase estimation, it is possible to account for the missing tones by appropriately scaling the phase differences. Note - for the purposes of this description, the terms

'tone' and 'carrier' are interchangeable, the latter term being preferred for generic multicarrier systems, the former used in the context of OFDM systems.

Two useful characteristics of the algorithm become apparent: (1) that noise effects may be partially or mostly mitigated by the tone-to-tone differential nature of the algorithm and (2) that the essential nature of the algorithm creates a weighting of timing estimates based on the likely validity of each estimate. These observations are explained in greater depth below.

One may think of the process of estimating timing offset as the differential detection of tone phase from one tone to its neighbor, in frequency. For QPSK, individual tone modulation can be removed from the estimate by raising the differentially detected phase to the 4^{th} power. Alternatively, each tone phase could be compared to the transmitted signal constellation and the complex conjugate of the nearest signal used to rotate the signal to the positive real axis. While noise and other impairments will occasionally create incorrect decisions about the transmitted signal and thus the received signal phase, these effects will average out across a number of estimates. As another alternative, if the correct transmitted data sequence were known in advance by the receiver, this information could be used to determine the expected phase of each tone. This information could be used as described to rotate the received signal to the positive real axis. To gain the signal to noise advantage of averaging the tone-to-tone estimates, it is possible to average the phase of each measurement. For N tones in an OFDM block, this approach requires the equivalent of N-1 arctangent calculations, which is prohibitively expensive in a real-time system. A preferred embodiment, which gives better performance under many channel conditions, is to separately average the in-phase and quadrature components of the processed constellation points and calculate the phase of *that* signal to determine the timing offset correction. FIG. 6 shows the constellation that results after differential-in-frequency

detection of the individual tones. FIG. 7 shows the same constellation when each signal point is raised to the 4^{th} power. Simulation results show that for reasonable amounts of timing offset, the phase angle of the resultant signal is directly proportional to the timing offset. The proportionality breaks down when the timing offset creates a very large rotation of the estimation signal. As the estimation signal approaches the (-1,0) point in the signal plane, there is an obvious 180 degree phase ambiguity, which could be addressed by processing the received signal instead of the 4^{th} power signal. There is an additional degradation in the system performance as the timing offset causes the FFT waveform to wander far outside the cyclic extension region into the windowed section of the data. Neither of these limitations are considered serious, since this timing estimation algorithm in intended to provide a steady state tracking control signal. Coarse initial timing acquisition (to within reasonable fraction of the OFDM block) is sufficient to get this timing recovery algorithm started.

- FREQUENCY OFFSET CORRECTION

Having addressed timing offset, a technique to estimate frequency offset is needed. FIG. 8 shows the OFDM signal phase versus frequency, this time with a frequency offset of $1/10^{th}$ the tone spacing. While the discrete QPSK phase levels are still evident, two effects can be seen: First, all the tones' phases are shifted uniformly on average. Second, there is a slow variation of phase across the OFDM spectrum, giving the appearance that the frequency offset is creating beat frequencies with the individual tones. By averaging across a number of tones, the second effect can be removed, leaving the phase offset on each tone as a predictor of frequency offset. As it was for timing offset, while noise and other impairments may affect individual tone phases, the systematic change in tone phase is present for all tone pairs and can be combined to obtain a robust estimate.

As before, assume that a series of samples, x_n are generated at the transmitter. With a frequency offset of Δf the received samples, r_n are:

$$r_n = x_n \cdot e^{j \cdot \frac{2 \cdot \pi \cdot n \cdot \Delta f}{N}}$$

or, combining this with the previous expression for the received tones,

$$R_n = \frac{1}{\sqrt{N}} \cdot \sum_{m=0}^{N-1} \left(x_m \cdot e^{j \cdot \frac{2 \cdot \pi \cdot m \cdot \Delta f}{N}} \right) \cdot e^{-j \cdot \frac{2 \cdot \pi \cdot n \cdot m}{N}}$$

As was used for estimating timing offset, frequency offset is similarly estimated by calculating the average phase of the ensemble of the received tones. Again, the 4th power signals may be used to remove the modulation. Alternatively, the other techniques described earlier may be employed to estimate the transmitter phase based on known or inferred transmit data. As was the case for timing offset, it is possible to calculate either the phase of the average signal or the average of the phases of the individual signals. For moderate amounts of frequency offset (less than 70% of the tone spacing), either technique works well. In fact, the lower complexity estimate (phase of the mean signal) tolerates larger frequency offsets well – more than 110% of the tone spacing. For typical OFDM systems, frequency offsets and Doppler shifts will generally be a fraction of the tone spacing.

- PHASE OFFSET CORRECTION

Finally, it is necessary to consider the effect of phase offset in the channel. The effect of the phase rotation can be expressed as:

$$r_n = x_n \cdot e^{j \cdot \Delta \phi}$$

So the received tones are given by:

$$R_n = \frac{1}{\sqrt{N}} \cdot \sum_{m=0}^{N-1} \left(x_n \cdot e^{j \cdot \Delta \phi} \right) \cdot e^{-j \cdot \frac{2 \cdot \pi \cdot m \cdot n}{N}}$$

The constant rotation factor can be factored out of the summation, indicating that a constant phase rotation of the baseband waveform results in a uniform rotation of each of the transformed tones.

A constant phase offset in the channel gives the same appearance to the received signal constellation that frequency offset produces. By measuring the rotation of the constellation and driving that parameter to zero by modifying the baseband rotator's phase updates, both frequency offset as well as phase rotation of the channel will be compensated for. Actually, in terms of the final signal constellation, the phase rotation caused by frequency and phase offset will not be the limiting factors – a system which uses differential (in time) detection of the signal phase is immune to fixed or (moderate amounts of) changing rotation. The most significant degradation caused by frequency offset is the inter-tone interference caused by the loss of orthogonality, which only requires that the frequency offset be approximated – phase lock is not essential.

VI. SIMULATION RESULTS

To determine the utility of the timing and frequency offset estimation algorithm, it is necessary to assess its performance in the presence of a variety of channel impairments. In particular, it is necessary to consider how the algorithm behaves in the presence of the following impairments:

- White Gaussian noise
- Frequency offset while making timing offset estimates
- delay spread/frequency selective fading

These impairments were simulated using Mathcad both individually and in various combinations. Results are presented below.

First, AWGN was simulated at varying signal to noise ratios. Even when the SNR was 6 dB, giving a 4^{th} power differential in frequency constellation as shown in FIG. 9, the timing offset estimator was able to track the timing offset based on one OFDM block. Table 1 shows some representative timing offset estimates at various SNRs and different timing offsets for the two measures (phase of mean and mean of phase).

TABLE 1

SNR (dB)	Timing offset (samples)	Mean of Phase Estimate (samples)	Phase of Mean Estimate (samples)
6	3	2.82	4.20
8	3	3.06	4.54
12	3	3.07	2.84
20	3	3.00	3.11
6	10	8.04	9.94
8	10	9.45	8.06
12	10	9.90	9.27
20	10	10.01	10.04
3	-22	-7.30	-19.20
8	-22	-20.7	-22.40
12	-22	-22.06	-21.33
20	-22	-21.96	-22.00
6	40[5]	24.01	42.37
8	40	20.55	39.17
12	40	33.92	39.64
20	40	39.97	39.94

It can be seen from these AWGN results that, even in the presence of poor SNRs, the algorithm tracks timing offset well, preserving the direction of adjustment and, even in the worst case, preserving the general magnitude of the adjustment needed. Again, it must be noted that these results are for single OFDM block processing. Averaging estimates over successive blocks improves the estimate in the presence of noise. At least in AWGN, simulation results show that the mean of the phase estimate tracks more closely to the actual timing offset than the phase of the mean estimate. While the former estimate is more costly, from a signal processing

perspective, with fewer tones' phases differentially compared and with multi-block averaging, it may be preferable to use this method under some circumstances.

Next, it is instructive to look at the effect that frequency offset has on timing offset estimation. As FIG. 8 suggests, frequency offset is unlikely to have much effect on the timing offset estimation. To verify this, the same simulations were conducted as were presented inTable 1, this time with various amounts of frequency offset. Representative results are listed in Table 2.

TABLE 2

SNR (dB)	Timing offset (samples)	Frequency Offset (fraction of tone spacing)	Mean of Phase Estimate (samples)	Phase of Mean Estimate (samples)
8	10	.1	8.89	8.04
8	10	.5	7.82	9.05
8	10	1.0	7.01	13.86
20	-22	.1	-22.02	-21.93
20	-22	.5	-21.94	-22.31
20	-22	1.0	-18.23	-23.53

While these numbers show the overall performance of the timing offset estimator, it is instructive to examine the signal constellation that the estimator is working with. FIG. 10 shows the effect on the differential in frequency constellation due to a large amount of simultaneous timing and frequency offset. It can be seen that, while the timing offset causes the constellation to "twist," the frequency offset creates a variation in amplitude (due to inter-tone interference) that causes the constellation points to create loci of points originating at (-1,0). In spite of this amplitude variation, the centroids of the twisted constellation points are essentially unchanged.

Finally, simulations of the timing offset estimator were run with delay spread. In one representative experiment, two equal rays with a spacing of 20.8 μsec (46 samples) were generated. With 10 samples of timing offset, it would be expected that the estimator would

indicate that 18 samples of correction was needed[1]. With a 10 dB SNR and .1 tone frequency offset, the timing offset estimator indicated 15.7 samples for the mean of the phase estimate and 19.8 samples for the phase of the mean estimate. As shown by the real-time DSP results, delay spread has minimal effect on the timing recovery algorithm.

The frequency offset estimator was next tried under simulated channel conditions. It is first worth noting, however, that it is not possible to estimate the frequency offset in the presence of timing offset. Timing offset creates an ever-increasing rotation of the tone signal constellation, making phase measurements meaningless. So, for the remaining simulations, it is assumed that either the timing offset has been adjusted to zero by adjusting the sample clock or the individual tones have been rotated by the proper amount to compensate for the timing offset.

VII. A PHYSICAL INTERPRETATION OF THE ALGORITHMS PERFORMANCE

We have observed from the above that (1) the noise perturbation of the signal phase does not create a significant problem for timing estimation and (2) the algorithm inherently provides a desirable weighting of timing estimates. These ideas are expanded in this section to provide some intuition about why the timing offset algorithm behaves as well as it does.

First consider the effect of noise. In FIG. 11, observe an arbitrary set of tone phases. It is assumed that $tone_i$ and $tone_{i+2}$ provide good estimates of the timing phase, but $tone_{i+1}$ is a noisier signal, giving a poorer estimate of timing phase. Since the tone phases are being differentially compared to determine the overall timing phase estimate, it can be seen that the positive bias in comparing $tone_i$ to $tone_{i+1}$ will tend to cancel the negative bias in comparing $tone_{i+1}$ to $tone_{i+2}$. Thus, as the simulations revealed, good timing estimates can be expected in rather poor SNR conditions. Further, since *all* of the tone phases have the *same* systematic bias due to timing

[1] With 10 samples of timing offset in the opposite direction to the 46 samples delay spread, the centroid of the

offset, the overall timing estimate will magnify this. Meanwhile, although analysis suggests that there is correlation between adjacent noise samples, in combination, the noise correlation across the entire band of frequencies is small and does not seem to disrupt the timing estimate.

The wireless delay dispersive channel that this OFDM system must endure creates frequency selective fading. From this, one might suspect that tones which are in the middle of a frequency null will have poor SNR and cannot be relied on to provide accurate phase estimates to drive the timing algorithm. Worse, the attenuation of one tone generally alters two phase estimates – if $tone_{i+1}$ is near an amplitude null, the phase estimate derived from comparing $tone_i$ to $tone_{i+1}$ as well as the phase estimate obtained from $tone_{i+1}$ and $tone_{i+2}$ may both be corrupted. Experimental results have shown remarkable performance in the presence of this frequency selective fading. Even when large clusters of tones are suppressed, the timing recovery algorithm performs well. Understanding why this was so provided additional insight into the power of this algorithm.

As shown, each tone is used to estimate the phase of its neighbor. This is done by multiplying the complex conjugate of the first tone with the second tone. This process preserves amplitude information. By collecting the sums of the real and imaginary parts separately, each differential tone pair adds to the phase estimate with an imaginary component proportional to the phase difference *and to the amplitude*. Further, each differential tone pair contributes to the real component proportionally to the amplitudes of the two tones. Thus, low amplitude, faded, and, probably noisy tones make a much smaller contribution to the overall timing estimate than do high amplitude, most likely clean tones do. The result is that frequency selective fading (i.e., a time dispersive channel) has little effect on the timing estimate. Further, since this algorithm

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channel response would be (46-10)/2=18 samples from the initial sampling point.

raises the differential signal to the 4^{th} power to remove QPSK modulation, any amplitude differences between tone pairs will be further accentuated.

VIII. LIMITATIONS AND PRACTICAL CONSIDERATIONS

In the preceding description, it has been assumed that all of the OFDM tones would be used to do timing estimation. With 189 active tones, this provides 188 differential phase estimates to be averaged. The signal processing, on a per tone pair basis, is moderate, but cannot be neglected. First, to compute the product of a tone with its neighbor's complex conjugate requires 4 real multiplies and two real adds. Then, to raise this signal to the 4^{th} power requires 12 real multiples and 3 adds, so the total number of operations is 16 multiplies and 5 adds. It is likely that this can be improved slightly, but this still will be a moderate amount of signal processing. If all 188 tone pairs are used, the receiver will obviously be in the best position to get a good timing estimate, but it is instructive to see just how few tone pairs were sufficient.

In the presence of noise and other impairments, one tone pair is not sufficient to allow the receiver to properly estimate the proper timing instant. With two or more tone pairs, the noise and interference effects described above are observed. As more and more tones pairs are used, the contribution of each individual tone pair's phase estimate is lessened so any applicable degradation of that estimate is likewise reduced.

In one embodiment of the invention, it is possible to average the phase difference between each tone pair to arrive at an overall phase estimate and then to derive synchronization information from that average phase. In the preferred embodiment of the invention, the real and imaginary components of the differentially detected difference between the tones are each summed to arrive at the real and imaginary parts of an overall correction vector. The phase of this overall correction vector is then calculated, representing the overall synchronization signal.

By performing the calculations in this manner a simpler algorithm results, with one arctangent calculation required instead of one arctangent calculation per tone pair.

The resulting invention provides a method for OFDM systems that is capable of separately estimating time and frequency offset in the presence of severe channel impairments, requiring no training sequences or other overhead. The algorithm is capable of deriving high-quality estimates on a single OFDM block or, with lower real-time overhead, could be used to track drift over a series of OFDM blocks.

Various illustrative examples of the invention have been described in detail. In addition, however, many modifications and changes can be made to these examples without departing from the nature and spirit of the invention.

CLAIMS

What is claimed is:

1. A data communications system, comprising:

a plurality of individually modulated transmission carriers;

one or more receivers, where each receiver has advance knowledge of relationships between phases of a transmitter's unmodulated carriers; and

a means to synchronize signals between the transmitter and receivers, based on an inherent structure in the frequency domain representation of the received waveform, without additional 'pilot' signals, synchronization patterns, or other special synchronization signals or waveforms.

2. The system of claim 1, where the transmitter's and receivers' signals' sample timing is being synchronized.

3. The system of claim 2, where the individually modulated transmission carriers are Orthogonal Frequency Division Multiplexed carriers.

4. The system of claim 2, where the frequency domain representation of the received signal is a form of the Fourier Transform.

5. The system of claim 4, where the form of the Fourier Transform is the Fast Fourier Transform.

6. The system of claim 2, where the structure of the frequency domain representation is the collective phase relationships between a plurality of individual carriers.

7. The system of claim 6 where the means to synchronize the timing of signals is based on computing the differences in phase between a plurality of individual carriers.

8. The system of claim 7, where the means to compute the differences in phase between individual carriers is by using a differential-in-frequency detection scheme.

9. The system of claim 8 where the differential-in-frequency detection scheme is to multiply a first carrier's complex representation by the complex conjugate of a second carrier's complex representation.

10. The system of claim 7 where the plurality of carriers used are adjacent in frequency.

11. The system of claim 7 where the plurality of carriers used is equally spaced but not adjacent.

12. The system of claim 7 where the plurality of carriers used may not be equally spaced but may be arbitrarily selected by the receivers.

13. The system of claim 6, where the plurality of carriers are used in combination to determine the synchronization, with the contribution of each phase estimate derived from the differential phase of each pair of carriers weighted according to the phase estimate's accuracy.

14. The system of claim 13 where the accuracy of each phase estimate is based on the amplitudes of the carriers used.

15. The system of claim 9 where, for each carrier pair, a first carrier's amplitude and phase, represented as a complex number in a Cartesian coordinate system, is multiplied by a second carrier's amplitude and phase, similarly represented, yielding a complex number representing the combined amplitude as well as the phase difference of the carrier pair; then calculating the vector sum of the plurality of the carrier pairs' combined amplitudes and phase differences; and finally using this vector sum to arrive at a timing synchronization signal.

16. The system of claim 15 where the carriers' modulating data signals are known by the receivers and can be used to determine the precise transmit carriers' phases.

17. The system of claim 15 where the carriers' modulating data signals are not known by the receivers, but can be estimated by attempting to demodulate the carriers and then using the derived modulating data to estimate the transmit carriers' phases.

18. The system of claim 15 where the carriers' modulating data signals are not known by the receivers, but where the effect of the modulation can be removed from the carriers without demodulating the carriers.

19. The system of claim 18 where the means to remove the carriers' data modulation is by raising the complex representation of the carrier amplitude and phase to an integer power.

20. The system of claim 19 where the modulation of the carriers is by N level phase modulation and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the N^{th} power.

21. The system of claim 20 where the modulation is Quadrature Phase Shift Keying and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the fourth power.

22. The system of claim 1, where the transmitter's and receivers' operating frequencies are being synchronized.

23. The system of claim 22, where the plurality of transmission carriers are Orthogonal Frequency Division Multiplexed carriers.

24. The system of claim 22, where the frequency domain representation of the received signal is a form of the Fourier Transform.

25. The system of claim 24, where the form of the Fourier Transform is the Fast Fourier Transform.

26. The system of claim 22 where the means to synchronize the operating frequency is based on computing the phases of a plurality of individual carriers.

27. The system of claim 26 where the plurality of carriers are used in combination to determine the synchronization with the contribution of each carrier weighted according to its accuracy.

28. The system of claim 27 where the accuracy of each carrier's contribution is determined based on the carrier's amplitude.

29. The system of claim 28 where, for each carrier, the carrier's amplitude and phase, represented by a complex number in a Cartesian coordinate system, is summed with the other carriers' complex representation to yield a vector sum, representing the composite amplitude and phase; and a means to use the phase of this composite vector to create a frequency synchronization signal.

30. The system of claim 29 where the carriers' modulating data signals are known by the receivers and can be used to determine the precise transmitter carriers' phases.

31. The system of claim 29 where the carriers' modulating data signals are not known by the receivers but can be estimated by attempting to demodulate the carriers and then used to estimate the transmit carriers' phases.

32. The system of claim 29 where the carriers' modulating data signals are not known by the receivers but where the effect of the modulation can be removed from the carriers without demodulating the carriers.

33. The system of claim 32 where the means to remove the carriers' data modulation is by raising the complex representation of the carrier amplitude and phase to an integer power.

34. The system of claim 33 where the modulation of the carriers is by N level phase modulation and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the N^{th} power.

35. The system of claim 34 where the modulation is Quadrature Phase Shift Keying and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the fourth power.

36. The system of claim 1 where the data communications system is a wireless data communications system.

37. The system of claim 36 where the system is subject to a variety of transmission channel impairments.

38. The system of claim 37, where the transmission channel impairments may include noise, interference, time sample misalignment, carrier frequency offset, or delay spread/frequency selective fading, singly or in combination.

39. The system described in claim 14 where, for each carrier pair, a first carrier's amplitude and phase, represented as a complex number in a Cartesian coordinate system, is multiplied by a second carrier's amplitude and phase, similarly represented, yielding a complex number representing the combined amplitude as well as the phase difference of the carrier pair; then calculating the vector sum of the plurality of the carrier pairs' combined amplitudes and phase differences; and finally using this vector sum to arrive at a timing synchronization signal.

40. A method for estimating time and frequency offset in an orthogonal frequency division multiplex system, comprising the steps of:

for any adjacent pair of tones R_i and R_{i+1}, measuring the differential phase from one tone to the next;

adjusting the sampling point to compensate so that the timing offset has the same phase difference between every pair of tones R_i and R_{i+1};

estimating the timing offset across all adjacent tone pairs in a burst; and

estimating the timing offset in the presence of signal impairments;

whereby the noise effects are mitigated by the tone-to-tone differential measurements, thereby creating a weighting of timing estimates based on the likely validity of each estimate.

41. The method of claim 40 wherein said impairments include frequency offset, noise, fading, delay spread/frequency selective fading.

42. A system for estimating time and frequency offset in an orthogonal frequency division multiplex system, comprising:

for any adjacent pair of tones R_i and R_{i+1};

means for measuring the differential phase;

means for adjusting the sampling point to compensate so that the timing offset has the same phase difference between every pair of tones R_i and R_{i+1};

means for estimating the timing offset across all adjacent tone pairs in a burst; and

means for estimating the timing offset in the presence of signal impairments;

whereby the noise effects are mitigated by the tone-to-tone differential measurements, thereby creating a weighting of timing estimates based on the likely validity of each estimate.

43. The system of claim 42 which further comprises said impairments including frequency offset, noise, fading delay spread/frequency selective fading.

44. In a data communications system, including a plurality of individually modulated transmission carriers, the method comprising:

storing information about relationships between a plurality of transmission carrier phases in their unmodulated states at one or more receivers; and

synchronizing signals between the transmitter and receivers, based on an inherent structure in the frequency domain representation of the received waveform, without additional 'pilot' signals, synchronization patterns, or other special synchronization signals or waveforms.

45. The method of claim 44, where the transmitter's and receivers' signals' sample timing is being synchronized.

46. The method of claim 45, where the individually modulated transmission carriers are Orthogonal Frequency Division Multiplexed carriers.

47. The method of claim 45, where the frequency domain representation of the received signal is a form of the Fourier Transform.

48. The method of claim 47, where the form of the Fourier Transform is the Fast Fourier Transform.

49. The method of claim 45, where the structure of the frequency domain representation is the collective phase relationships between a plurality of individual carriers.

50. The method of claim 49, where the synchronizing of the timing of signals is based on computing the differences in phase between a plurality of individual carriers.

51. The method of claim 50, where computing of the differences in phase between individual carriers is by using a differential-in-frequency detection scheme.

52. The method of claim 51 where the differential-in-frequency detection scheme is to multiply a first carrier's complex representation by the complex conjugate of a second carrier's complex representation.

53. The method of claim 50 where the plurality of carriers used are adjacent in frequency.

54. The method of claim 50 where the plurality of carriers used is equally spaced but not adjacent.

55. The method of claim 50 where the plurality of carriers used may not be equally spaced but may be arbitrarily selected by the receivers.

56. The method of claim 49, where the plurality of carriers are used in combination to determine the synchronization, with the contribution of each phase estimate derived from the differential phase of each pair of carriers weighted according to the phase estimate's accuracy.

57. The method of claim 56 where the accuracy of each phase estimate is based on the amplitudes of the carriers used.

58. The method of claim 52 where, for each carrier pair, a first carrier's amplitude and phase, represented as a complex number in a Cartesian coordinate system, is multiplied by a second carrier's amplitude and phase, similarly represented, yielding a complex number representing the combined amplitude as well as the phase difference of the carrier pair; then calculating the vector sum of the plurality of the carrier pairs' combined amplitudes and phase differences; and finally using this vector sum to arrive at a timing synchronization signal.

59. The method of claim 58 where the carriers' modulating data signals are known by the receivers and can be used to determine the precise transmit carriers' phases.

60. The method of claim 58 where the carriers' modulating data signals are not known by the receivers, but can be estimated by attempting to demodulate the carriers and then using the derived modulating data to estimate the transmit carriers' phases.

61. The method of claim 58 where the carriers' modulating data signals are not known by the receivers, but where the effect of the modulation can be removed from the carriers without demodulating the carriers.

62. The method of claim 61 where the means to remove the carriers' data modulation is by raising the complex representation of the carrier amplitude and phase to an integer power.

63. The method of claim 62 where the modulation of the carriers is by N level phase modulation and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the N^{th} power.

64. The method of claim 63 where the modulation is Quadrature Phase Shift Keying and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the fourth power.

65. The method of claim 44, where the transmitter's and receivers' operating frequencies are being synchronized.

66. The method of claim 65, where the plurality of transmission carriers are Orthogonal Frequency Division Multiplexed carriers.

67. The method of claim 65, where the frequency domain representation of the received signal is a form of the Fourier Transform.

68. The method of claim 67, where the form of the Fourier Transform is the Fast Fourier Transform.

69. The method of claim 65 where the means to synchronize the operating frequency is based on computing the phases of a plurality of individual carriers.

70. The method of claim 69 where the plurality of carriers are used in combination to determine the synchronization with the contribution of each carrier weighted according to its accuracy.

71. The method of claim 70 where the accuracy of each carrier's contribution is determined based on the carrier's amplitude.

72. The method of claim 71 where, for each carrier, the carrier's amplitude and phase, represented by a complex number in a Cartesian coordinate system, is summed with the other carriers' complex representation to yield a vector sum, representing the composite amplitude and phase; and a means to use the phase of this composite vector to create a frequency synchronization signal.

73. The method of claim 72 where the carriers' modulating data signals are known by the receivers and can be used to determine the precise transmitter carriers' phases.

74. The method of claim 72 where the carriers' modulating data signals are not known by the receivers but can be estimated by attempting to demodulate the carriers and then used to estimate the transmit carriers' phases.

75. The method of claim 72 where the carriers' modulating data signals are not known by the receivers but where the effect of the modulation can be removed from the carriers without demodulating the carriers.

76. The method of claim 75 where the means to remove the carriers' data modulation is by raising the complex representation of the carrier amplitude and phase to an integer power.

77. The method of claim 76 where the modulation of the carriers is by N level phase modulation and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the N^{th} power.

78. The method of claim 77 where the modulation is Quadrature Phase Shift Keying and the data modulation is removed by raising the complex representation of the carrier amplitude and phase to the fourth power.

79. The method of claim 44 where the data communications system is a wireless data communications system.

80. The method of claim 79 where the system is subject to a variety of transmission channel impairments.

81. The method of claim 80, where the transmission channel impairments may include noise, interference, time sample misalignment, carrier frequency offset, or delay spread/frequency selective fading, singly or in combination.

82. The method of claim 57 where, for each carrier pair, a first carrier's amplitude and phase, represented as a complex number in a Cartesian coordinate system, is multiplied by a second carrier's amplitude and phase, similarly represented, yielding a complex number representing the combined amplitude as well as the phase difference of the carrier pair; then calculating the vector sum of the plurality of the carrier pairs' combined amplitudes and phase differences; and finally using this vector sum to arrive at a timing synchronization signal.

ABSTRACT OF THE DISCLOSURE

The synchronization technique invention uses inherent characteristics of the frequency domain representation of the data symbols. By computing a differential-in-frequency function across a large number of OFDM tones, robust estimates of time and frequency offset can be easily obtained. The technique also allows the system designer to directly trade performance in the presence of channel impairments against signal processing complexity. Analysis and simulation have shown good performance in the presence of noise and channel delay dispersion, impairments that are the harshest in a wireless environment.

Prior techniques for OFDM synchronization have focussed on the time domain representation of the signal. Those that have recognized the translation of time and frequency offset to the frequency domain have not considered the systematic modification of the signal by the offsets.

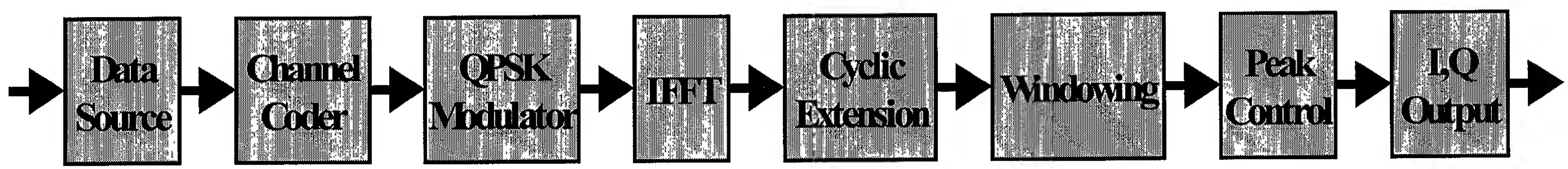

FIGURE 1

4233_1

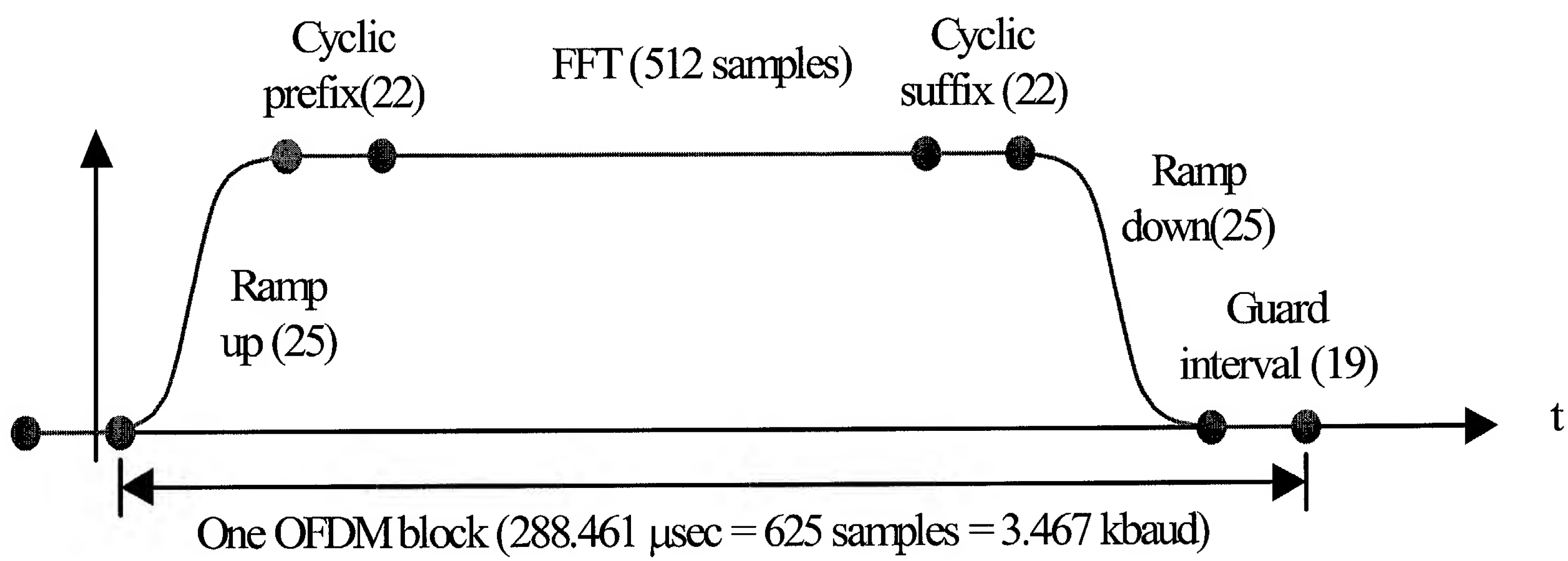

FIGURE 2

4233_1

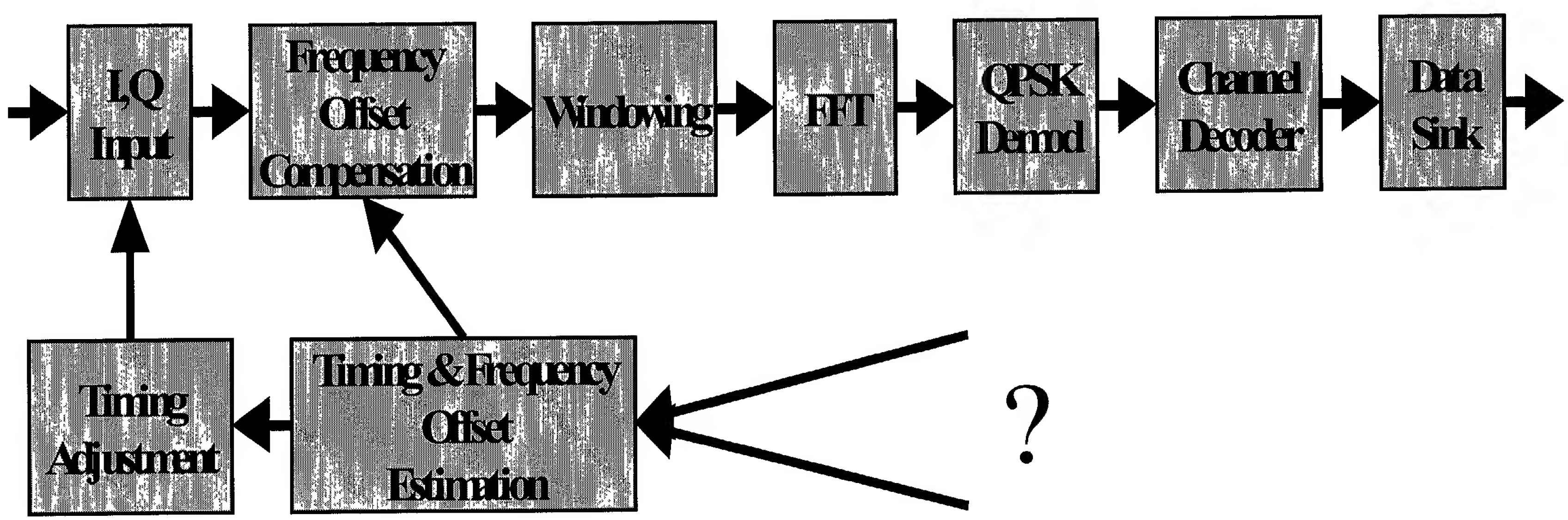
I,Q Input
Frequency Offset Compensation
Windowing
FFT
QPSK Demod
Channel Decoder
Data Sink
Timing Adjustment
Timing & Frequency Offset Estimation
?

FIGURE 3

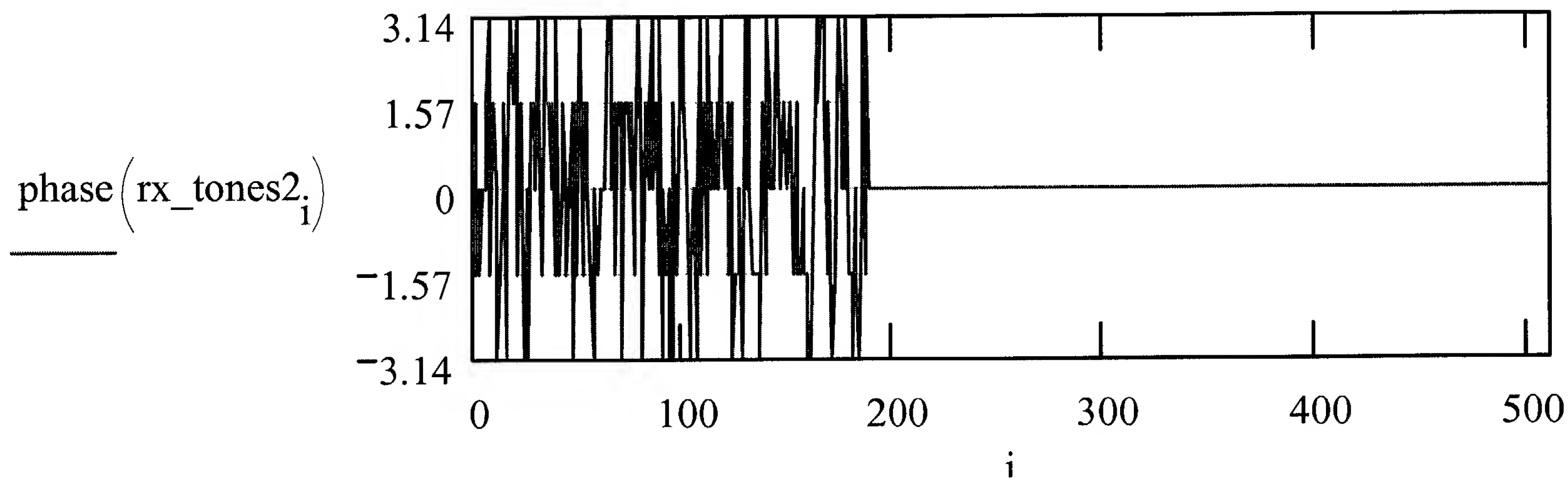

FIGURE 4

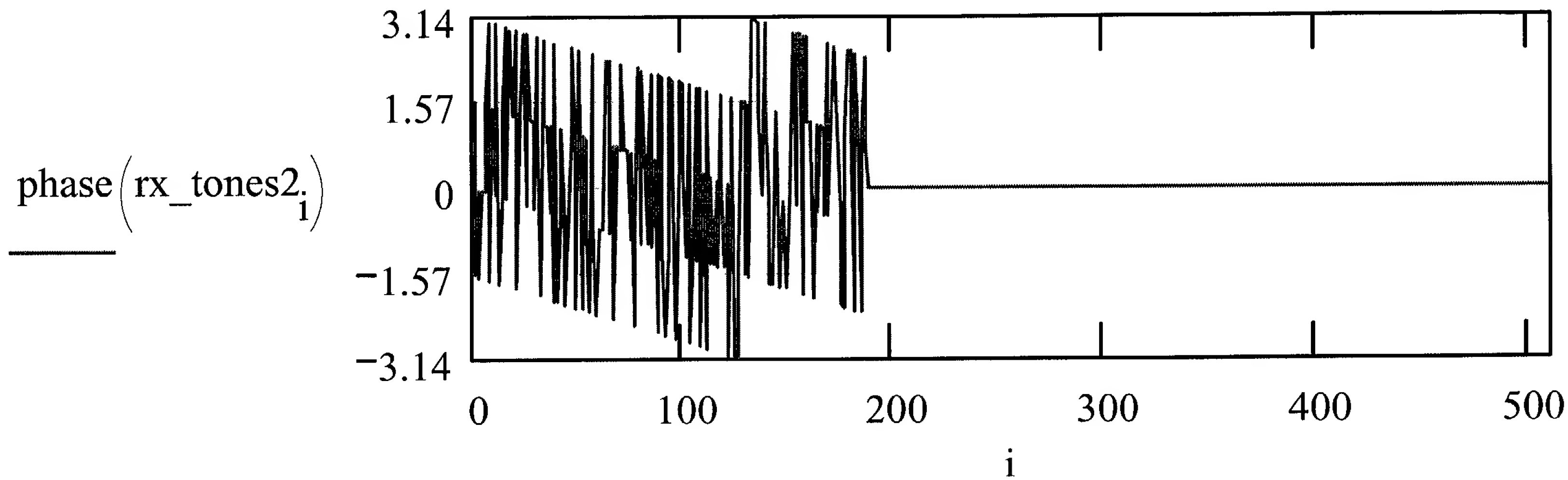

FIGURE 5

4233_1

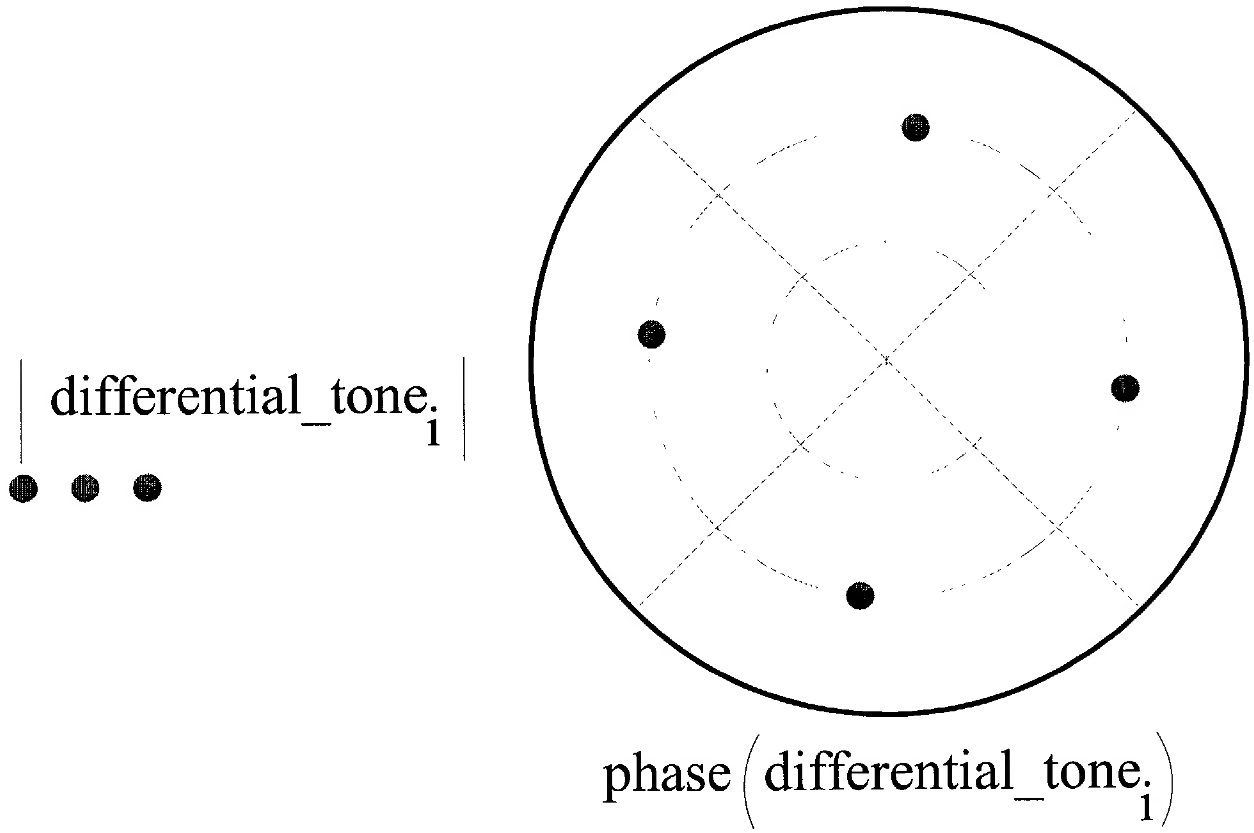
$\left| \text{differential\_tone}_1 \right|$
$\text{phase}\left(\text{differential\_tone}_1\right)$

FIGURE 6

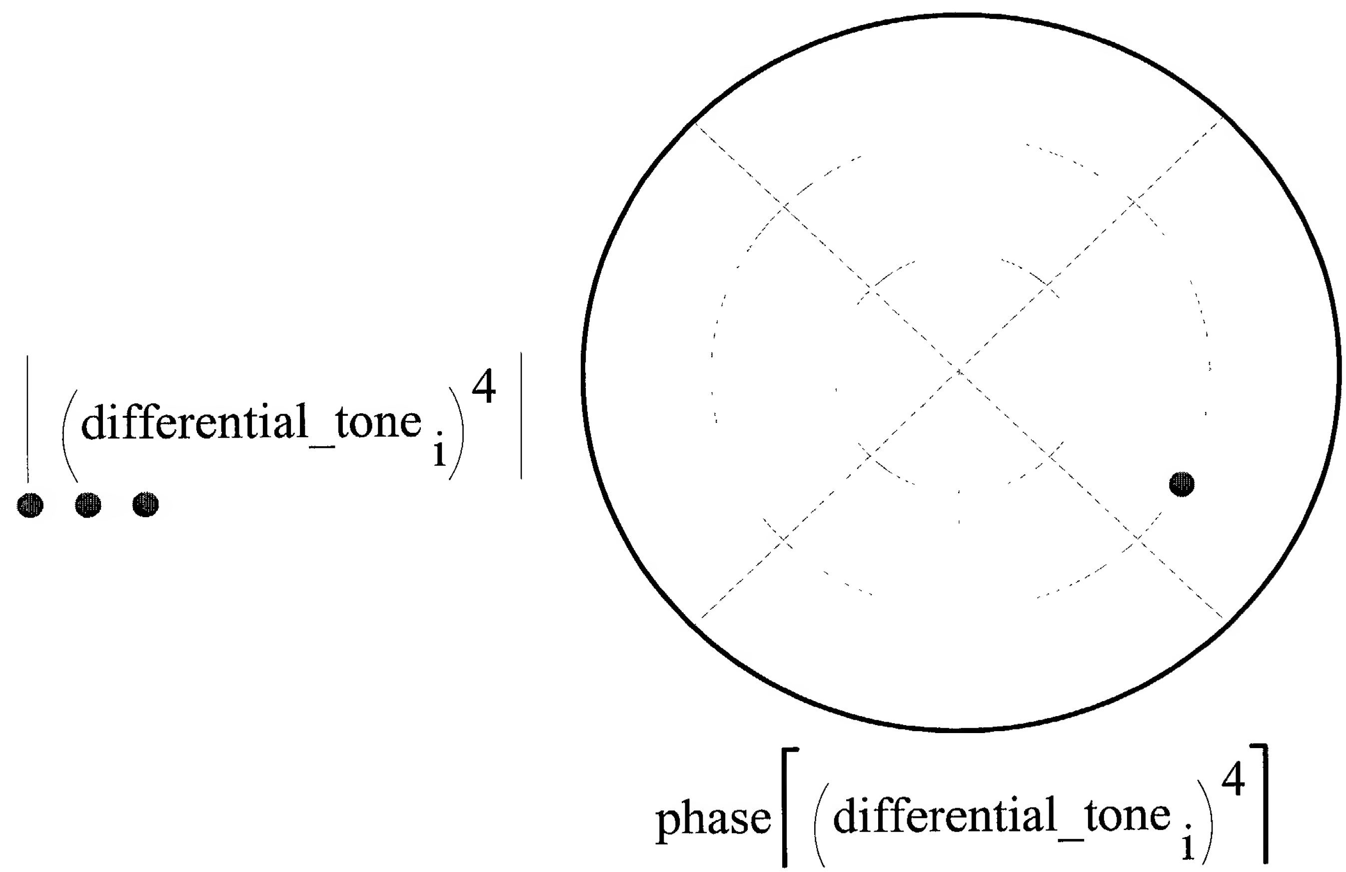

FIGURE 7

4233_1

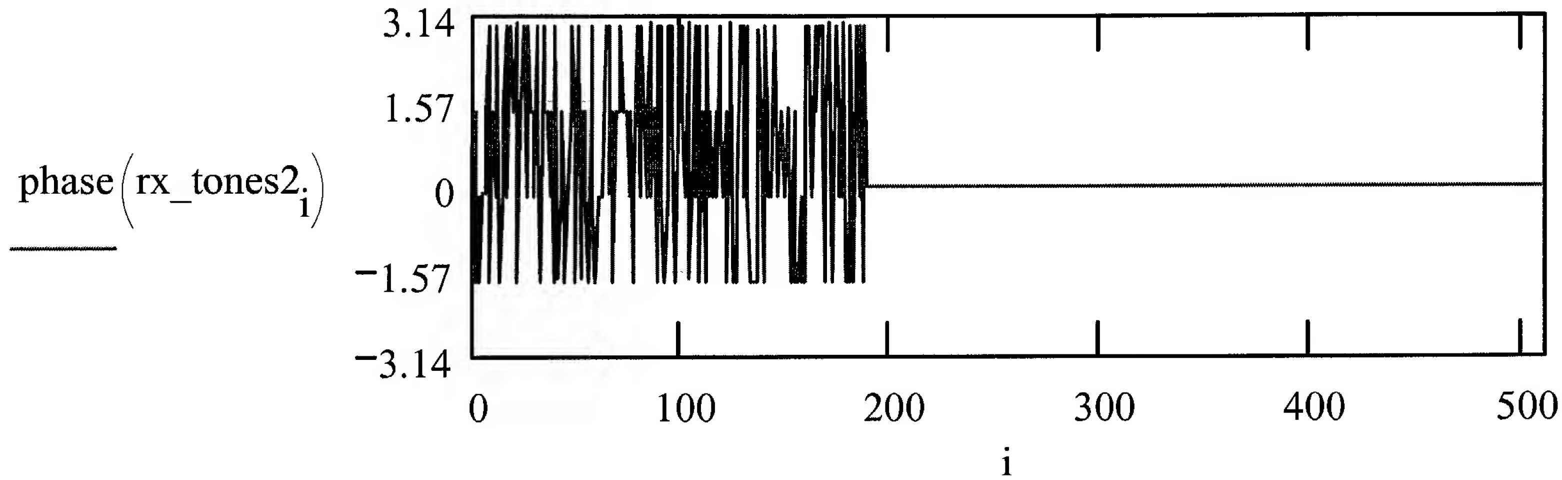

FIGURE 8

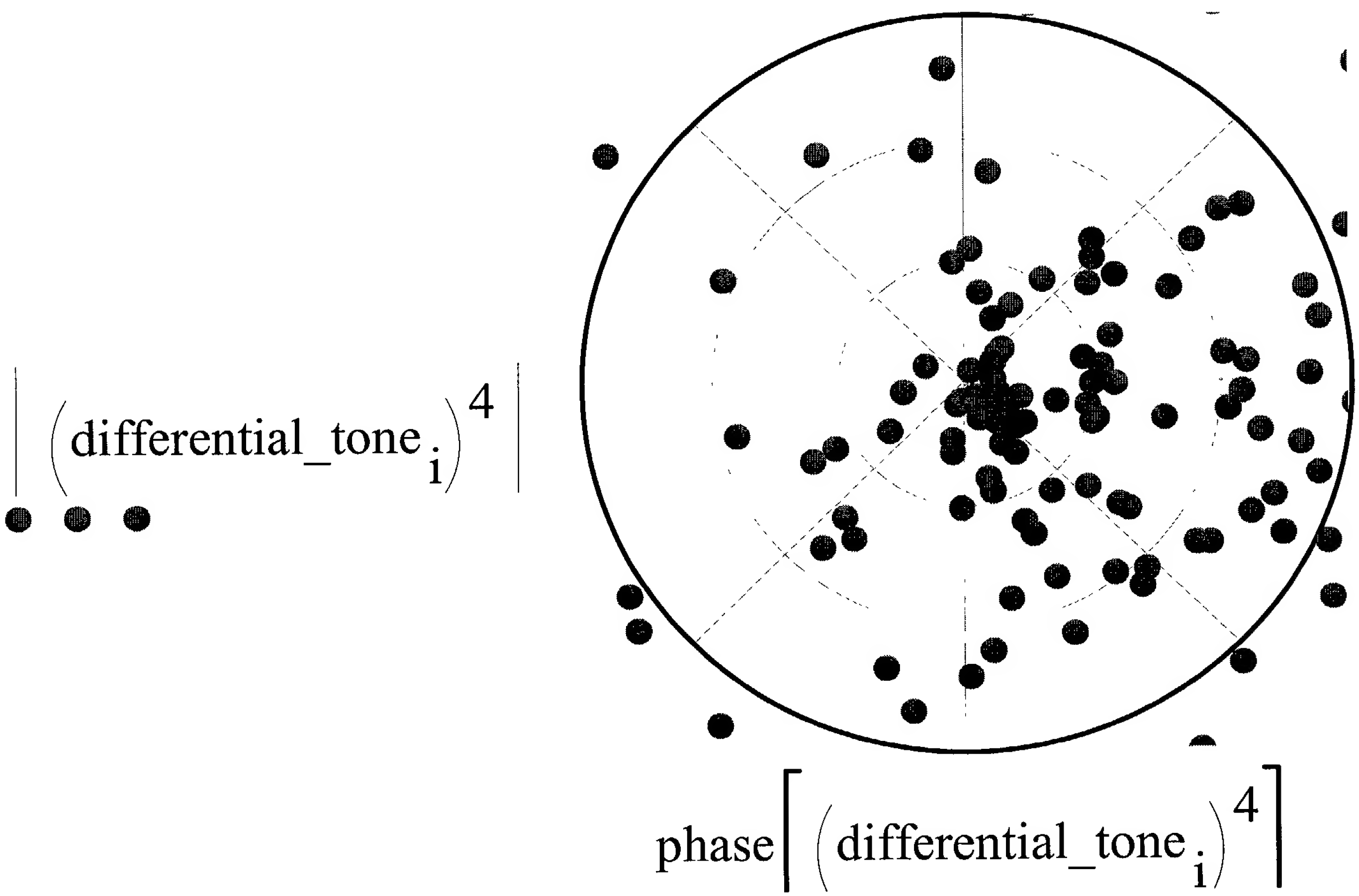
$\left|\left(\text{differential\_tone}_i\right)^4\right|$
$\text{phase}\left[\left(\text{differential\_tone}_i\right)^4\right]$

FIGURE 9

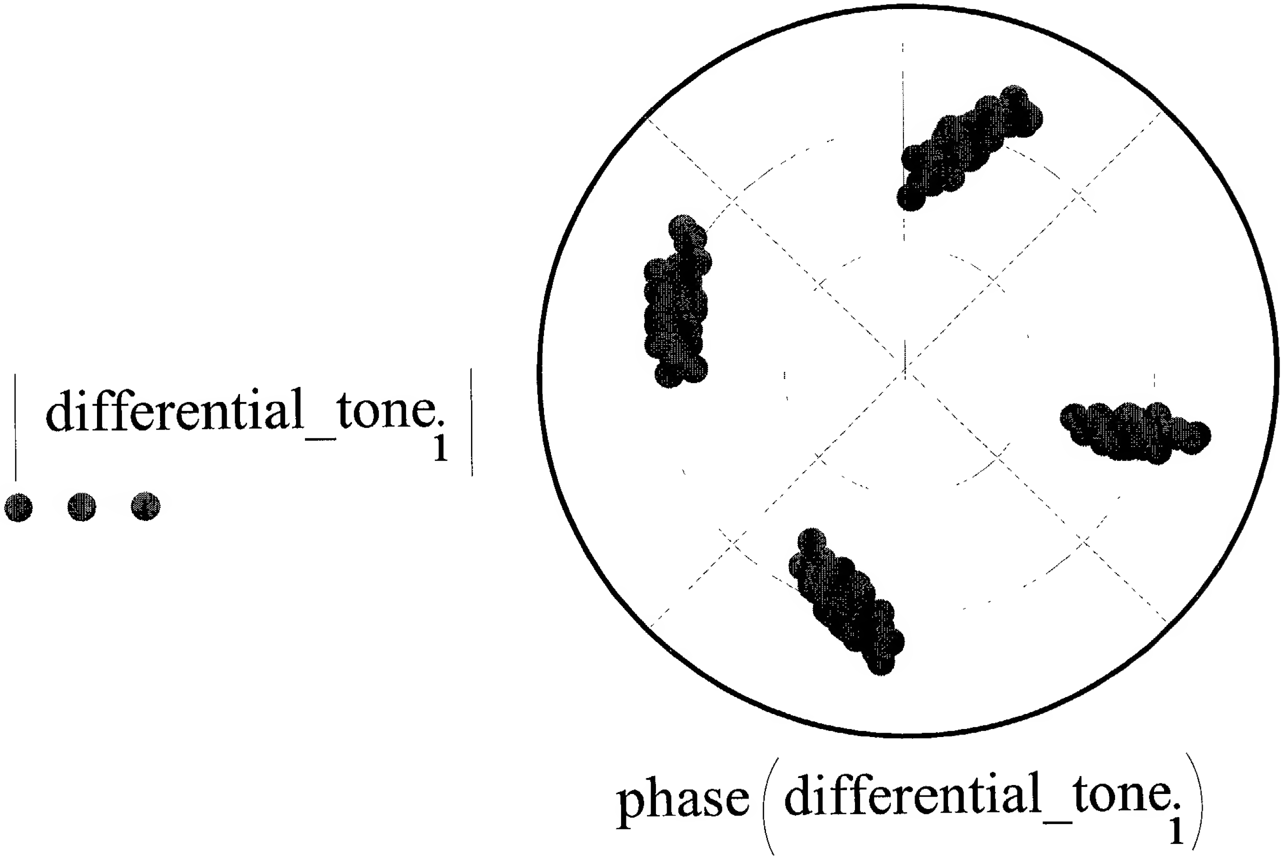

FIGURE 10

4233_1

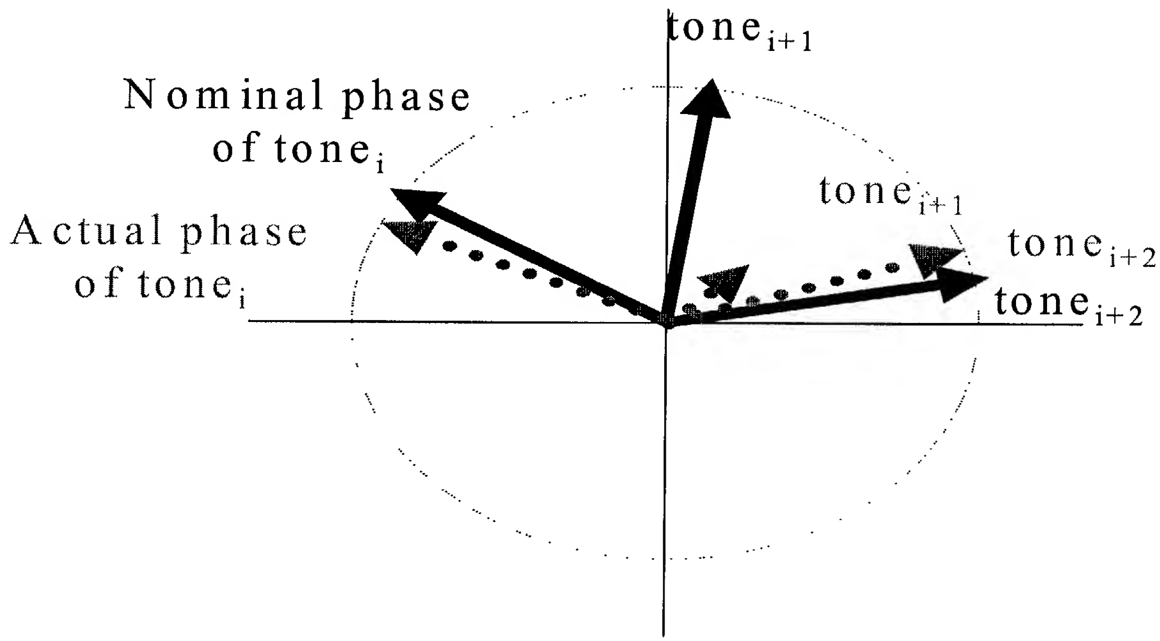
tone$_{i+1}$
Nominal phase of tone$_i$
Actual phase of tone$_i$
tone$_{i+1}$
tone$_{i+2}$
tone$_{i+2}$

FIGURE 11